Technical Comments

Potential Energy of a Normal Pressure Field Acting on an Arbitrary Shell

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A PROOF has been given by Cohen[1] that a nonuniform continuous normal pressure field acting on an arbitrary shell has potential energy if, at the edge of the field, some conditions are existing. The purpose of this Note is to show that there is also another expression for the potential energy π, consistent with the variational expression given in Ref. 2.

The pressure field shall be defined by the function $p(x^i)$ where x^1 and x^2 are curvilinear coordinates of the undeformed shell reference surface s, and x^3 is a normal coordinate measured from s. In the notation of Ref. 1, the virtual work of the pressure p per unit deformed area is given by

$$A = -\int\int_s\{p\varphi_\alpha\,\delta u^\alpha + [p(1+e_\alpha{}^\alpha)+u^\alpha p;_\alpha + u^3 p_{,3}]\,\delta u^3\}\,ds \quad (1)$$

where

$$\varphi_\alpha = b_{\alpha\beta}u^\beta - u^3;_\alpha$$
$$e_{\alpha\beta} = \tfrac{1}{2}(u_\alpha;_\beta + u_\beta;_\alpha) + b_{\alpha\beta}u^3 \quad (2)$$

Applying the divergence theorem for a smooth surface one has

$$\tfrac{1}{2}\int\int_s\{pu^3\,\delta u^\alpha;_\alpha - pu^\alpha\,\delta u^3;_\alpha\}\,ds =$$
$$-\tfrac{1}{2}\int\int_s\{(pu^3);_\alpha\,\delta u^\alpha - (pu^\alpha);_\alpha\,\delta u^3\}\,ds +$$
$$\tfrac{1}{2}\int_\Gamma(pu^3\,\delta u^\alpha - pu^\alpha\,\delta u^3)n_\alpha\,d\Gamma \quad (3)$$

Where Γ is the edge contour of s and n_α is a unit outward normal in s to Γ. Substituting Eq. (3) into Eq. (1) and making use of the symmetry of $b_{\alpha\beta}$ gives the result

$$\delta A = -\delta\pi + \tfrac{1}{2}\int_\Gamma(pu^3\,\delta u^\alpha - pu^\alpha\,\delta u^3)n_\alpha\,d\Gamma \quad (4)$$

where

$$\pi = \tfrac{1}{2}\int\int_s\{p\varphi_\alpha u^\alpha + [p(2+e_\alpha{}^\alpha)+u^\alpha p;_\alpha + u^3 p_{,3}]u^3\}\,ds \quad (5)$$

Therefore, the conclusion is that if the expression $\tfrac{1}{2}\int_\Gamma(pu^3\,\delta u^\alpha - pu^\alpha\,\delta u^3)n_\alpha\,d\Gamma$ is zero, then the pressure field has the potential energy π.

The difference between the functional π of Eq. (4) and the functional given by Ref. 1, Eq. (6) is

$$-\tfrac{1}{2}\int\int_s(pu^3u^\alpha);_\alpha\,ds = -\tfrac{1}{2}\int_\Gamma(pu^3u^\alpha)n_\alpha\,d\Gamma \quad (6)$$

In the special case of a pressure field on shell of revolution with a coordinate system of Ref. 2, Appendix B, Eq. (5) reduces to

$$\pi = \tfrac{1}{2}\int_\xi\int_\theta\{p\phi_\xi u_\xi + p\phi_\theta u_\theta + [p(2+e_\xi+e_\theta) + (\partial p/\partial\xi)u_\xi + (\partial p/r\partial\theta)u_\theta]w\}\,rd_\xi\,d\theta \quad (7)$$

where ϕ_ξ, ϕ_θ, e_ξ, e_θ, u_ξ, u_θ and w are defined in Ref. 2, Appendix B.

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References

[1] Cohen, G. A., "Conservativeness of a Normal Pressure Field Acting on a Shell," *AIAA Journal*, Vol. 4, No. 10, Nov. 1966, p. 1886.

[2] Budiansky, B., "Notes on Nonlinear Shell Theory," *Transactions of the ASME, Journal of Applied Mechanics*, Vol. 35, No. 2, June 1968, pp. 393–401.

Errata

Photoelastic Model Analysis of Sandwich Beams

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THE captions of the two photographs should be interchanged and the photographs should be turned upside down.

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Index categories: Properties of Materials; Structural Composite Materials (Including Coatings); Optimal Structural Design.